pencils of conics on coresidual quadruples in such a way that in the projectivity the three degenerate members of the two pencils correspond.

[11] Abstracts I and II, these PROCEEDINGS, **7**, 1921 (245, 334).

[12] Coble, *Trans. Amer. Math. Soc.*, **14**, 1913 (261).

[13] Coble, *Trans. Amer. Math. Soc.*, **17**, 1916 (358).

ON IRREGULARITIES IN THE VELOCITY CURVES OF SPECTROSCOPIC BINARIES

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The spectrographic velocity curves of several Cepheid variable stars, and of a few other stars not yet definitely known to be variable, show puzzling deviations from simple elliptical orbital motion. These deviations have hitherto been explained by superposing upon a primary elliptical velocity curve a secondary oscillation (generally circular) whose period must be *precisely* one-half or one-third the period of the primary. Such treatment has produced fairly good agreement with the observed velocity data. There is, however, a similarity in the location of the nodes of the primary and secondary curves which savors of artificiality. Moreover, it is very difficult, if not impossible, to devise reasonable and stable multiple systems, or tidally distorted stars, which shall produce such anomalies of one-half or one-third the period of the system.

My failure to construct a dynamically reasonable tidal or other model giving such oscillations in an exact submultiple of the period has led me to the attempt to represent the observational data by means of a purely elliptical velocity curve, plus a *single* oscillation or "hump." This new method of treatment is of considerable interest and, if substantiated by future more accurate spectrographic data, will have an important bearing upon some of the many theories of Cepheid variation, the most puzzling feature of which, as is well known, is the essential synchronism of maximum light with maximum velocity of approach.

Limitations of space make it impossible to give in this paper the curves which have been derived for these stars under the hypothesis of secondaries of a submultiple of the period; these curves will be found in the original papers, references to which are given at the close. In cuts I–VI, shown in Figures 1 and 2, the original spectrographic velocities are plotted without change as given by the respective observers, the dots representing three-prism results, and the open circles those derived from one-prism

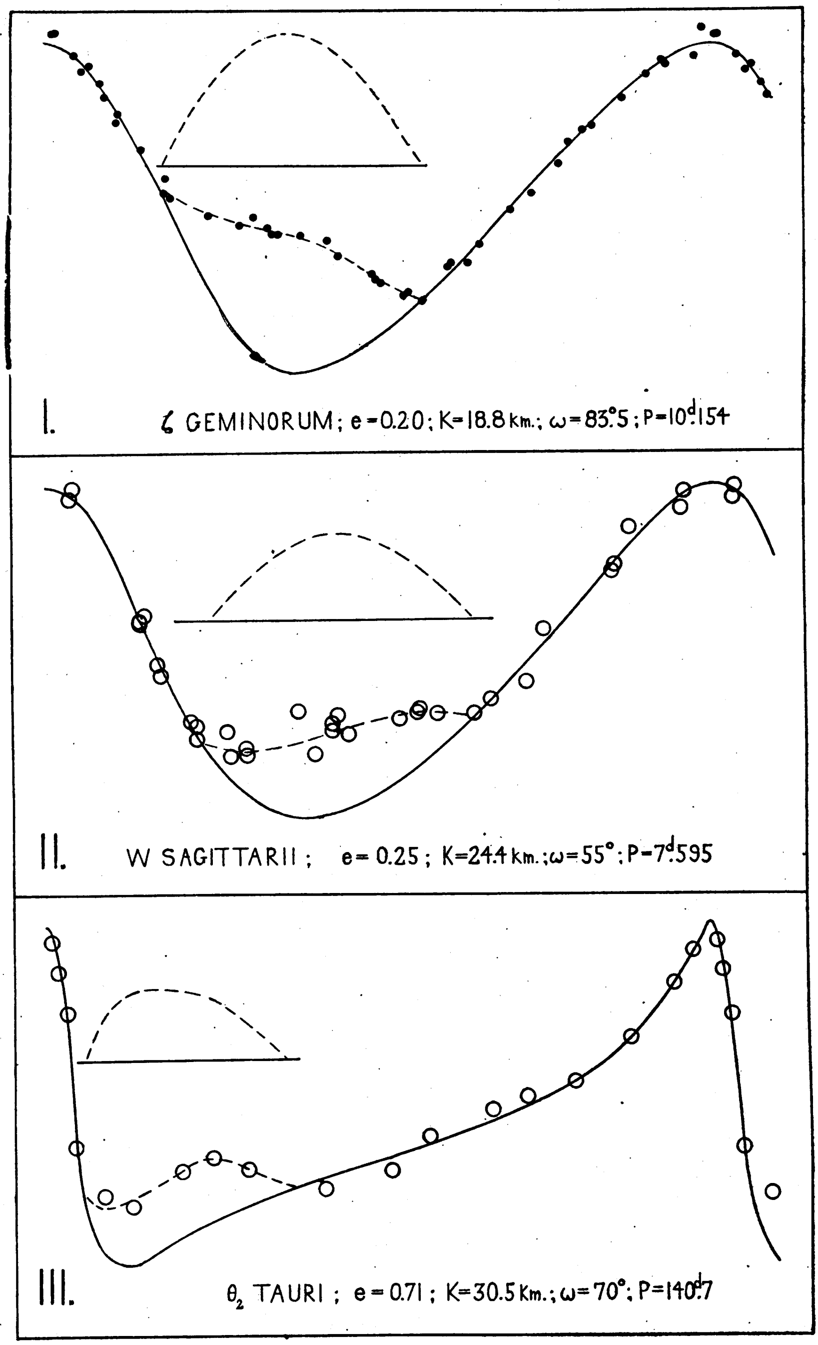
I.
ζ GEMINORUM; e = 0.20; K = 18.8 Km.; ω = 83°.5; P = 10d.154
II.
W SAGITTARII; e = 0.25; K = 24.4 Km.; ω = 55°; P = 7d.595
III.
θ2 TAURI; e = 0.71; K = 30.5 Km.; ω = 70°; P = 140d.7

FIGURE 1

observations. In all the cuts, maximum velocity of approach is the lower peak of the curve, with which the epoch of maximum light in the Cepheids is closely synchronous.

Legends for Fig. 1

I. ζ Geminorum.[1] A Cepheid variable. Observations of high accuracy, obtained by Campbell with the Mills three-prism spectrograph. Recent observations by Mr. Jacobsen at Lick Observatory show the hump in the same place after twenty-three years. The agreement of the present curve, in its undistorted portion, is rather better than from an elliptical orbit with a secondary oscillation of one-third the main period.

II. W. Sagittarii, a Cepheid variable.[2] Observations of moderate accuracy, by Curtiss with the one-prism Lick spectrograph. The agreement in the undistorted portion of the curve is about as good as that secured through a secondary of one-half the main period. It will be noted that the hump is, as before, nearly symmetrical with the velocity minimum.

III. θ_2 Tauri.[3] The observations are fairly accurate, being normal places by Plaskett based on one-prism observations, with the inclusion of a number of three-prism values. The agreement in the undistorted portion of the curve seems better than with a secondary of one-half the period. Not known to be variable.

Though there is a verisimilitude in the humps obtained by this new method of treatment, all occurring close to the time when the star is approaching us most rapidly, an explanation may be as difficult as in the case of the hypothesis of secondaries of an exact submultiple of the period. Two possible explanations may be tentatively given.

1. These humps might be due to a blend phenomenon, for which we have numerous analogies in the curves of a number of spectroscopic binaries. This explanation seems unlikely, first, because blending or line doubling has been sought for in vain in several of the cases treated; and, secondly, the slight but definite asymmetry of these humps is unlike a blend effect.

2. One theory of Cepheid variation, due in its original form to Duncan,[8] explains the synchronism of light maximum and highest velocity of approach, as well as the periodic changes in spectral type, by the assumption that the large and tenuous star whose light we observe is rotating about a darker star in a slightly resisting medium. This medium "brushes back" the rare outer atmosphere of the star on its advancing face. Such a "brushing back" would have the effect of a quasi-depression, most noticeable as the star is coming directly toward us; if the star is large, this quasi-depression may still be convex as viewed from the interior of the star. Such depressions would give rise to "humps" like those drawn in the velocity curves. Their effective depths would be of the order of 1,000,000 km. for ζ Geminorum, and 500,000 km. for *W* Sagittarii, quantities which are not unreasonable if these stars are large. This explanation is somewhat more difficult in the cases of θ_2 Tauri and φ Persei, for here the quasi-depression would need to have an effective depth of the order of 10,000,000 km. Even this would not be unreasonable were these stars as large as Betelgeuze. Other difficulties in this theory cannot be treated here.

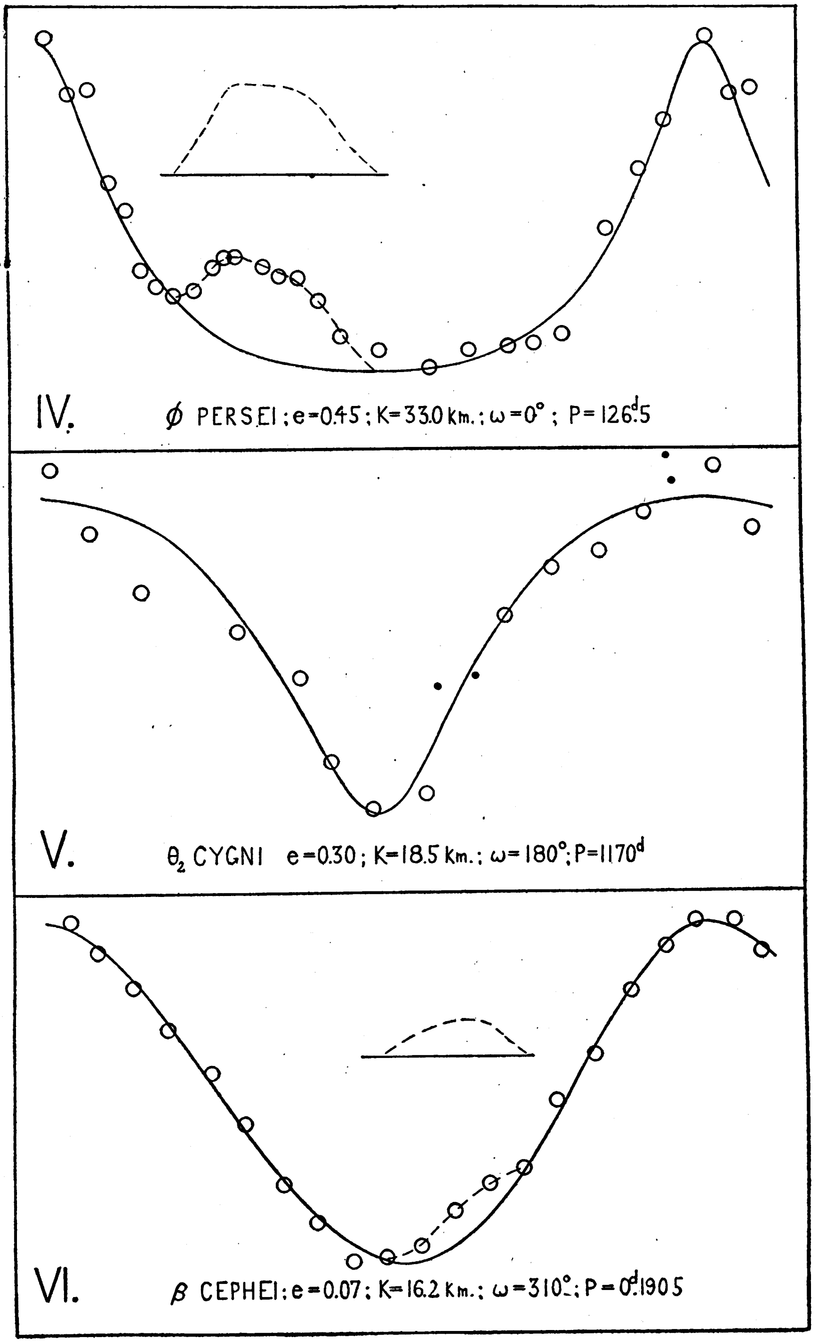
IV.
ϕ PERSEI: e=0.45; K=33.0 Km.; ω=0°; P=126.d5
V.
θ2 CYGNI e=0.30; K=18.5 Km.; ω=180°; P=1170d
VI.
β CEPHEI: e=0.07; K=16.2 Km.; ω=310°; P=0.d1905

FIGURE 2

Legends for Fig. 2

IV. φ Persei.[4] This star is not known to be a variable. The observations are of moderate accuracy, being normal places from one-prism observations by Jordan at Allegheny. It is a star of very complex and erratic behavior. Cannon[5] (observations not shown) postulated a secondary of one-half the main period, but it is impossible to satisfy Jordan's data in this manner.

V. θ_2 Cygni.[6] Not known to be a variable. The observations are of moderate accuracy, being normal places from one-prism observations by Cannon at Ottawa, with the inclusion of four Lick three-prism values. The observations are grouped almost precisely as would be expected were there a blend effect, but such an effect was searched for without result. Cannon's primary has an eccentricity of 0.18, and he introduces a secondary of one-third the main period. I cannot satisfy these observations with a single hump, as in the cases given before. It is of interest to see that an entirely different orbit from that of Cannon shows some measure of agreement. More observations are urgently needed on this star, particularly just after maximum velocity of recession, to decide on the character and reality of its deviations.

VI. β Cephei.[7] A Cepheid variable of unusually short period $0^{d}.19$ Crump's observations with the Detroit Observatory one-prism are normal places and of rather high accuracy. My curve does not satisfy these places quite so well as does his orbit, which postulates a secondary of one-third the main period, with a semi-amplitude of but 1.25 km.

One purpose of this paper will be fulfilled if those possessing instruments of adequate power will accumulate many more observations of the light and velocity curves of the Cepheids; only with such assistance will it ever be possible to solve the many puzzles which they present.

[1] Campbell, *Astrophys. J.*, **13**, 1900 (90).
[2] Curtiss, *Lick Obs. Bull.*, **3**, 1904 (62).
[3] Plaskett, *Publ. Dom. Obs.*, **2**, No. 2, 1915.
[4] Jordan, *Publ. Allegh. Obs.*, **3**, 1913 (31).
[5] Cannon, *J. R. A. S. Canada*, **4**, 1910 (195).
[6] Cannon, *Astrophys. J.*, **47**, 1918 (193).
[7] Crump, *Publ. Detroit Obs.*, **2**, 1915 (144).
[8] Duncan, *Lick Obs. Bull.*, **5**, 1908 (82).

ON THE FORM OF THE DISTRIBUTION LAW OF STELLAR VELOCITIES

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In the mathematical theory of stellar statistics we need, among others, an analytical expression for the distribution of cosmic velocities. From a theoretical point of view the simplest and most natural form of this "Velocity Law" is the Maxwellian form

$$F(v)dv = \frac{4h^3}{\sqrt{\pi}} e^{-h^2v^2} v^2 dv$$